J. Fluid Mech. (1982), *vol.* 116, *pp.* 283–296
Printed in Great Britain

Flow beneath a stagnant film on water: the Reynolds ridge

By JOHN C. SCOTT

Fluid Mechanics Research Institute, University of Essex,
Colchester CO4 3SQ, Essex, England

(Received 15 June 1981)

Surface-active material is present in most naturally occurring water samples, and it naturally diffuses steadily to free surfaces, where it both reduces the surface tension and gives the surface elastic properties which enable it to resist compression. When the water flows so that the surface layer is trapped and compressed against a fixed shallow-draught barrier the film material makes the surface incompressible, and flow beneath the barrier forms a viscous boundary layer under the film. The stresses associated with this boundary layer are found to distort the surface in the region of the leading edge of the film, giving rise to a phenomenon which is commonly observed in nature and which has been called the Reynolds ridge. This paper describes experimental work on the measurement of the ridge, and compares the results with a theoretical model due to Harper & Dixon. Good agreement is indicated.

1. Introduction

When surface-active material is accumulated by a flowing stream against a shallow-draught barrier, it tends to immobilize the surface, resisting compression and dilatation: liquid flowing freely towards the barrier encounters the edge of a static, apparently incompressible, surface and is retarded by the viscous boundary layer which forms beneath the stationary film. The pressure associated with this retardation in the boundary layer causes the surface in the vicinity of the leading edge of the film to be raised, and this is observed visually as a very small single ridge on the surface – the Reynolds ridge. Because surface-active material is almost always present in natural streams, the Reynolds ridge is quite commonly observed, and it has been reported frequently in the scientific literature.

The ridge may readily be seen by careful examination of many of the water-surface flow situations common in both nature and the laboratory. When a twig or stem of grass is caught in the surface of a fresh-water stream, acting as a trap for incoming floating material, the flow beneath the resulting surface film often gives rise to a ridge, seen as a fine, often fluctuating thread, some 50 to 150 mm upstream of the obstruction. The ridge usually marks the boundary between regions of greater or lesser calm, the film-covered region nearer the obstruction offering greater suppression of the wind- and turbulent-flow-excited surface disturbances than does the freely moving region. A steady wind may also cause a ridge at the leeward end of a closed pool of water, by virtue of the circulation it generates. In the laboratory, instances are similarly common. A particular example known to the author is found in a tank used for washing photographic materials, the steady incoming fresh-water flow maintaining a strong circulation which compresses the film against one side of the tank.

The earliest recorded observations appear in the writings of Henry D. Thoreau, who described natural occurrences of the phenomenon on three separate occasions, in his diaries for 1854, 1858, and 1859. These diaries were, however, not published until 1902, and the first widely published account appears to be due to Langton (1872). It is not known whether Osborne Reynolds' (1881) description follows from Langton's report, or whether it represents an independent discovery. However, Reynolds did describe both of the alternative forms of the ridge – the other being seen when a thin surface-active oil layer spreads unhindered on a clean water surface – and he demonstrated their equivalence. Reynolds' interest in the phenomenon has resulted in its being named after him (McDowell & McCutchen 1971). Rayleigh (1890) also demonstrated the spreading-oil version of the ridge.

Other published observations of the Reynolds ridge, a surprising number of which appear to have been independent discoveries, include those by Satterly (1919), Thompson (1919), Burdon (1926), Edser (1926), Satterly & Turnbull (1929), Woog (1931), Schmidt (1936), Stansfield (1936), Hall (1936), Burdon (1949), Bell (1954), Satterly (1956), Merson & Quinn (1965), Sellin (1968), Mockros & Krone (1968) and McCutchen (1970). It is interesting to note that one of these accounts (Edser 1926) was in a textbook of physics. Almost all of these papers are simply descriptive accounts of the phenomenon and the circumstances surrounding its occurrence, although some authors did attempt some form of explanation.

Reynolds was perhaps the first author to appreciate that an oil film spreads on water through the action of the higher surface tension of the clean part of the surface, rather than through repulsive forces between molecules in the film. However, the boundary-layer concept of representing the viscous forces beneath the film layer, which act to retard the film motion and which provide the stress needed to oppose the spreading, was not available to him at that time.

The first adequate description of the physical mechanism of the Reynolds ridge, in terms of the pressure distribution associated with the leading edge of the laminar boundary layer, is due to McCutchen (1970), who has also provided the clearest published illustrations of the phenomenon to date. The first quantitative theory of the Reynolds ridge was given by Harper & Dixon (1974), and the present experimental results, which are believed to be the first detailed measurements of the ridge, will be compared with the theoretical results of these authors.

2. Observation of the Reynolds ridge

The Reynolds ridge is visually elusive, and is by no means easily accessible to precise measurement. This section will discuss the relation between the true shape of the ridge and what is observed using particular observation techniques. Such a discussion is needed because it is very easy to make quite plausible misinterpretations of the evidence that comes from certain of the observation techniques that have been used. Such misinterpretations have indeed appeared in the literature.

2.1. *Direct visual observation*

The direct visual observation of the Reynolds ridge gives the distinct impression of a 'line', 'thread', 'rib' or 'ridge' lying on the surface. More detail than this, however, is extremely difficult to deduce. The ridge is obviously very small, and seems to be less than 1 mm high, but even this observation is less a direct estimate than an expression of inability to compare what is seen against an objective scale. The main problem in directly observing a smooth water surface lies in the fact that it either specularly reflects or refracts incident light, rather than scattering light randomly like an unpolished solid. In the general case, when the pattern of the incident light is not strictly controlled, what is seen can be very confusing, and it is not clear whether what is being observed is a ridge (upwards), a furrow (downwards), or a step (difference in level).

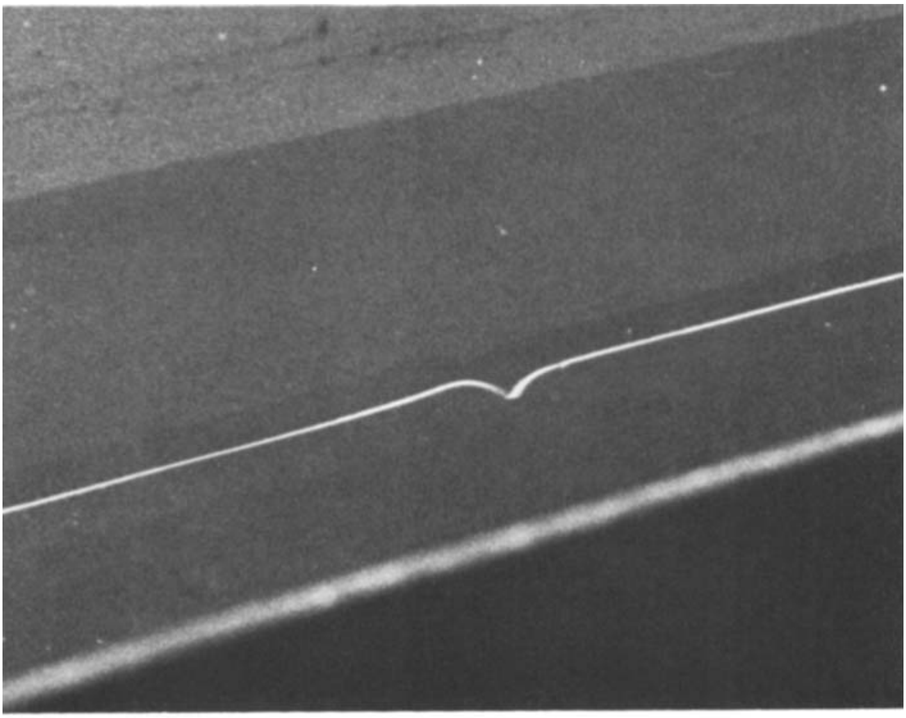
One visual observation which tends to rule out the possibility of a furrow is that the distorted surface can bring light from an elevated distant source to a single line focus below the surface, indicating a surface shape that is concave downwards. It is not possible, however, to discriminate in this way between ridges and steps, as the erroneous conclusions of Sellin (1968) demonstrate (see § 2.3).

2.2. *The observation of reflected or refracted straight lines*

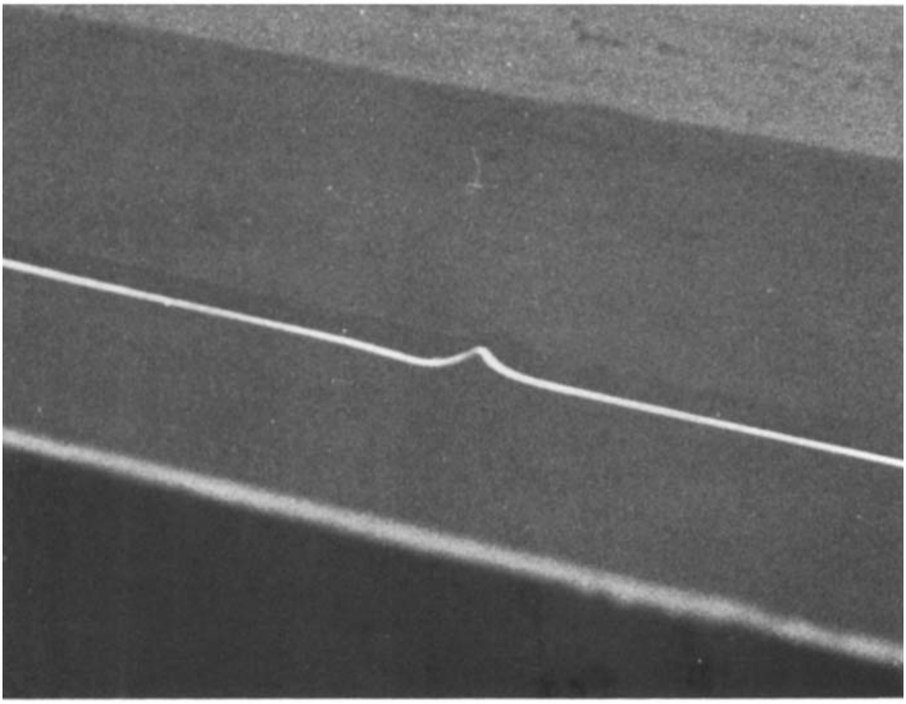
By simplification and strict control of the incident light it is often possible for more confident deductions to be made about what is being seen. McCutchen (1970) shows good pictures of below-surface grid patterns refracted by Reynolds ridges, and he correctly deduces that the shapes of the image distortions observed in this way give a rough idea of the slope variation along the stream direction. This deduction appears to have been based on intuition rather than analysis, for calculations by Gilbert & Scott (1980), of the effect of a straight-line surface shape on the image of a reflected straight-line light source (essentially the same arrangement used by McCutchen, although his work involved the observation of a line grid refracted through the surface), indicate that the deduction process is by no means simple. In the general case an observed image does not uniquely define the surface slope, and Gilbert (1980) shows a striking example of this non-uniqueness.

In practice, however, it is found that there are certain surface shapes for which deductions may be made with some confidence, and for which the intuitive approach of McCutchen proves to be broadly justifiable. The Gilbert & Scott analysis indicates that the non-uniqueness problem may be removed by making two or more static observations under different geometrical conditions, and a human observer can make a large number of complementary observations simply by moving his head. It is found possible, using mainly intuitive reasoning together with some appreciation of the precise analytical results, to derive a rule-of-thumb which works well for certain distorted surfaces, including the Reynolds ridge.

This rule-of-thumb may be illustrated using the pictures of the Reynolds ridge shown in figure 1. These photographs show the distorted line images seen when the observer looks obliquely at the reflection, in the water surface, of a long thin-line light source held parallel to the flow direction and just above the water surface. Both of the photographs were taken using a standard 55 mm lens on a Pentax camera, and the two observation positions were in the same horizontal plane, on the same side of the channel, looking down at the water surface, directly towards the Rynolds ridge, observation being from upstream looking downstream and downstream looking upstream, respectively. The internal surfaces of the channel were painted matt black, and the only other feature visible in these pictures is the nearer edge of the flow channel, diffusely reflecting light from the line source.



(a)



(b)

FIGURE 1. Photographs of the Reynolds ridge visualized using a long thin straight-line light source reflected in the water surface. The photographs were taken at the same height above the water surface: (a) looking from upstream towards the ridge; (b) looking from downstream towards the ridge. The apparatus is the same as that shown in figure 3. The water is flowing from left to right, and the bright line seen in the foreground is the top edge of the near side of the 128 mm wide channel. The length of the surface observed in these pictures is 300–400 mm.

The shape of the observed image distortion in this arrangement is seen to be approximately inverted by the change of observation point from upstream to downstream. Of the three surface-shape parameters which might be considered to have a significant effect on the observed image – the vertical displacement, the surface slope and the surface curvature – only the slope is expected to give an image distortion that is an odd function of the observation direction. It may thus be intuitively reasonable to deduce that the slope variation may have at least a major effect on the image distortion in the present case.

In a case such as this, the following line of reasoning is found to be valid. When visual observation is from a given direction, the image is observed to be displaced upwards if the corresponding part of the surface is inclined upwards towards the observer, and displaced downwards if the surface slopes downwards away from the observer. Obviously, when the observer changes his position from one side of the surface distortion to the other side, any part of the surface originally inclined upwards now appears inclined downwards, and the image is indeed seen to change accordingly.

2.3. *Schlieren observations*

The only quantitative data on the Reynolds ridge that has been published to date is that of Sellin (1968), who used a schlieren visualization technique. In this technique (Holder & North 1963), a parallel light beam is brought to a precise focus, and if an opaque stop is placed at this focus then the light beam is completely blocked. It is arranged that the parallel light beam, before being focused on to the stop, is allowed to pass through the water surface, and a camera is placed so that any light that does pass the stop position (for example if the stop were removed) will form a sharp image of an object in the water-surface plane. In the normal state, therefore, with the stop in place, the camera will record no image, but, if there are localized disturbances at the water surface which refract light past the fixed stop, then these disturbances will be made visible by the light they have deflected, and they will appear as sharply defined light patches.

Sellin's experiments used annular aperture stops, which pass only the light deflected between two angular limits. A series of twelve such stops was used, covering a range of upper and lower angular limits. There are two main disadvantages associated with annular stops: their rotational symmetry means that it is impossible to distinguish light deflected in one direction from light deflected in the opposite direction, and the finite radius of the central opaque area means that a deflection has to be greater than the smallest central disk radius for it to be detected at all. Thus, while the present results (to be reported in § 5) indicate clearly the existence of an extended gently sloping region upstream of the ridge, this was almost completely missed by Sellin's apparatus, because in the ridges he observed the surface slope was usually less than the *c.* 0·01 rad sensitivity of the smallest stop he used. Traces of the region may perhaps be discerned in figure 5 of Sellin (1968).

Similarly, although the present work clearly shows regions of opposite slope, Sellin reported slopes inclined in one direction only, which led him to the hydrodynamically untenable conclusion of a downwards surface 'step'. It is apparent that schlieren measurements must be treated with extreme caution in this case.

3. The theoretical model

An insoluble surface-active material will spread on the clean surface of a stationary body of water until the surface tension becomes everywhere uniform. The value of the surface tension ultimately produced will depend on the properties of the material, on the quantity of the material, and on the available surface area, and the surface tension will in general decrease as the surface concentration of material increases.

The surface is thus generally elastic in nature, dilatational strains (in-plane expansions and contractions) being opposed by surface-tension changes, and the surface will therefore appear to the underlying liquid to be a flexible, elastic boundary. Certain surface-active materials may well modify the detailed elastic properties of the contaminated surface, as a result of their surface rheological properties, but the dilatational elasticity of contaminated surfaces is common to all surface-tension-reducing contaminants. For this reason, the exact composition of the surface film is of little consequence for the production of the Reynolds ridge, determining only the total spatial extent of the compressed film resulting from a given quantity of material. The surface-tension variation in the film will be determined by the stresses exerted on it by the flowing water, although for different films a different variation of surface concentration will be needed to give the necessary variation of the surface tension.

If the underlying viscous liquid flows relative to this elastically restrained surface it will develop a laminar boundary layer (figure 2), which will exert a tangential compressive stress on the surface film. Consequently, the surface-active material present will be automatically rearranged to produce gradients of surface tension that balance the induced tangential stress. Mockros & Krone (1968) presented measurements of the surface-tension gradients associated with the boundary layer, and Krantz (Anon. 1974) has incorporated this effect into an elegant technique for measuring low levels of water contamination.

In a system whose surface-active material content is fixed, a film confined by a surface barrier will adjust to form an incompressible layer of finite extent, with a surface-tension variation which is in equilibrium with the tangential boundary-layer stress. Water at the clean surface of the approaching flow will suddenly encounter the immobile stagnant surface and be forced below it into the boundary layer.

Harper & Dixon (1974) used essentially this model to compute the surface shape associated with the Reynolds ridge, except that they assumed the presence of a low level of contamination in the free water surface approaching the stagnant film. The present observations and the work of Krantz (private communication, 1973), indicate that this restriction is not necessary, and that the Reynolds ridge persists even at the lowest achievable contamination levels. The model also assumes that the surface tension is constant in the region of the ridge. This assumption is reasonable because it is the surface-tension gradient which opposes the compression of the film, and the tension itself can be expected to change only slightly in the immediate region of the ridge.

By matching the flow conditions upstream and downstream Harper & Dixon obtained the following expression for the surface profile $H(X)$ associated with the leading edge of the film:

$$H(X) = \frac{2\beta}{\pi^{\frac{1}{2}}} \int_0^\infty \frac{\cos(t^2 X + \frac{1}{4}\pi)}{1 - mt^2 + t^2}\, dt,$$

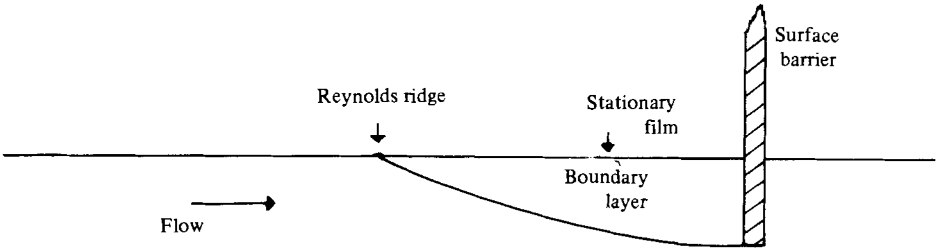


FIGURE 2. The geometry of the Reynolds-ridge system. Water flowing from left to right is forced beneath the stagnant film, forming a boundary layer. The Reynolds ridge is formed at the leading edge of the stagnant film.

where the dimensionless height H and the position X relative to the leading edge are scaled by the capillary constant $\gamma = (\sigma/\rho g)^{\frac{1}{2}}$. Here,

$$m = \left(\frac{\rho U^4}{\sigma g}\right)^{\frac{1}{4}}, \quad \beta = 0{\cdot}8604m(\nu/U\gamma)^{\frac{1}{2}},$$

where U is the stream velocity, σ is the surface tension, g is the acceleration due to gravity, ν is the kinematic viscosity, and ρ is the liquid density (in S.I. units).

The theoretical model makes the assumption that the leading edge is everywhere perpendicular to the flow, and that the surface behind it is immobile. This is not quite what is observed in practice. In a channel of finite width the flow variation across the channel produces a curved leading edge, concave upstream, and a circulation is observed in the surface behind the ridge. In the experiments reported here, this took the form of a relatively slow motion downstream along the channel centreline and a symmetrical backflow nearer the channel walls. This secondary flow persisted for some way into the stagnant film, eventually ceasing where the film became visibly present and relatively rigid with respect to shearing motions.

The theory breaks down for flow velocities greater than that corresponding to $m = 2$, which has the same value, for a given surface tension, as the minimum velocity of deep-water capillary–gravity waves. What is observed in practice at this point is that, as predicted by Harper & Dixon, the ridge forms a pronounced wavy structure upstream, the downstream part of the ridge, on the film-covered side, retaining the same general shape as for lower velocities.

4. The present work

4.1. *The flow channel*

Although it would have been preferable in these experiments to use water that was completely free from surface-active materials, tap water was used, principally because such a large volume was required, the difficulties associated with preparing such large volumes of clean water being very great (Scott 1979). The consequent problem of not knowing the state of cleanliness of the surface was dealt with by arranging that as the water flowed towards the confined surface film it was skimmed upstream by a fixed shallow-draught barrier. The surface was completely renewed at this barrier, and there was little time for the diffusion of significant quantities of material to the fresh surface before the water reached the leading edge of the observed stagnant film.

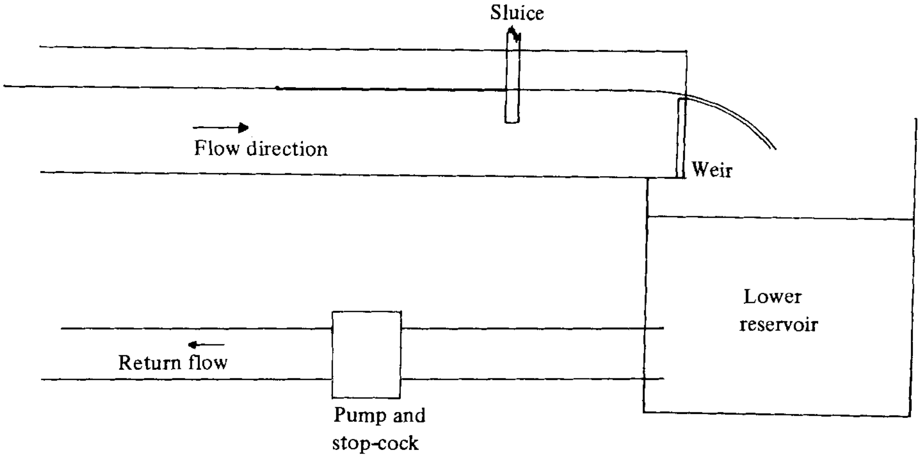


FIGURE 3. Diagram of the flow-channel apparatus used to observe the Reynolds ridge.

The films observed in the experiments were in all cases composed of naturally occurring materials already present in the tap water, and these were allowed to accumulate for at least 20 min before observations began. The relative unimportance of the exact chemical nature of the material making up the film is discussed in §3. At the time of a measurement the film was always apparently stable, neither increasing nor decreasing in extent over a time scale many times that required for a single observation of the surface shape.

The films observed were in a form of equilibrium between the surface-active material being accumulated at the leading edge of the film and that being lost by desorption from the more-compressed regions downstream. Because natural films are composed of a large number of different chemical species, the composition of the film would be expected to change along the compressed surface, the less-surface-active, less-tenacious species being lost the sooner. Thus, in addition to the secondary circulation described in §3 there was also a steady downstream motion in the film, although this was very much smaller than the stream velocity. If, during the course of an extended series of experiments, a film was found to become inconveniently long as the result of steady accretion, then part of the film was removed from the downstream end using absorbent tissue paper.

The apparatus used is shown diagrammatically in figure 3. It had two open reservoirs, connected by an open channel of rectangular section, width 128 mm, and also by a closed return-flow pipe including a stop-cock and a constant-speed electric pump. The entry into the channel from the upper reservoir was furnished with a smoothly finished flare, and the flow was observed to be laminar and reasonably steady. The channel was terminated downstream by a weir of adjustable height, and the surface barrier against which the film accumulated was a small removable sluice gate, whose height could be finely adjusted with screws and which could be pressed tightly into the channel at any position. The sluice gate allowed a gap of up to 20 mm for the passage of water.

The flow velocity was set by adjusting the return-flow stop-cock, the height of the weir, and the height of the sluice, and the velocity was measured by timing between 10 and 20 small fragments of thin tissue paper as they moved centrally with the flow between two fixed points that were well clear of both the first surface barrier and the beginning of the film.

4.2. *The surface-slope measurement*

A simple laser-beam deflection technique was used to measure the surface-slope variation associated with the Reynolds ridge (figure 4). The deflection of a laser beam reflected from the surface was detected by an optical system consisting of a cylindrical lens and a light-sensitive potentiometric device (Scott 1974). This gives an analogue voltage output proportional to the deflected light beam position. The cylindrical lens ensured that only beam deflections along the channel flow direction were measured.

The lens/detector system was fixed rigidly to the metal case of the horizontally mounted 0·5 mW He–Ne laser, and the light beam was deflected downwards to the surface at an angle about 5° from the vertical in the plane perpendicular to the flow direction. The whole apparatus (laser and optical system) was mounted on a sturdy photographic tripod equipped with a horizontally panning head, so that the laser beam could be scanned smoothly over the surface in the centre of the channel. The scanning arcs involved were of sufficiently small angle that their departure from straight lines was not significant, and the beam position was monitored continuously using a 150 mm long linear-motion potentiometer connected between the optical system and the flow channel by a mechanical linkage.

The slope-variation measurements thus involved the recording and calibration of two analogue electrical signals: one associated with the position of the laser beam on the water surface and the other representing the deflection of the reflected laser beam by the surface. These two analogue signals were processed in identical d.c. amplifier units, each with the facility of d.c. offset. The offset facility was found helpful because the 'central' positions of both potentiometric detectors gave non-zero outputs. Recording both signals on a storage oscilloscope was made relatively simple by using the d.c. offsets to zero both outputs at a convenient position on the water surface relative to the Reynolds ridge, before scanning the beam over the region of interest. After each scan of the apparatus over the ridge the screen of the storage oscilloscope, bearing the surface-slope-variation information, was photographed with a fixed-position Pentax 35 mm camera using a 55 mm $f/2{\cdot}8$ Takumar lens.

Calibration of the apparatus was done *in situ.* Pieces of opaque tape, of known width and separation, mounted on glass, were used to calibrate the position sensor, the taped glass being held steady in the plane of the water surface during a recording scan of the laser beam. The slope output trace recorded by scanning the laser beam over the tape gave a distinctive response at the tape edges, allowing the horizontal scale of the oscilloscope picture to be determined. The light-beam deflection measurement was calibrated in a similar manner, using a glass Fresnel biprism held in the surface. A scan over this prism gives a step change in output corresponding to the known small relative inclination of its two faces. These calibration procedures have the advantage that the response of the measuring electronics is checked quickly without needing to change the disposition of the apparatus, and the effect of ambient light on the light beam detection system can thus be assessed without difficulty. Ambient-light effects on the solid-state beam-position-sensing device were avoided by shrouding the apparatus in an opaque black cloth and avoiding strong lighting in the laboratory.

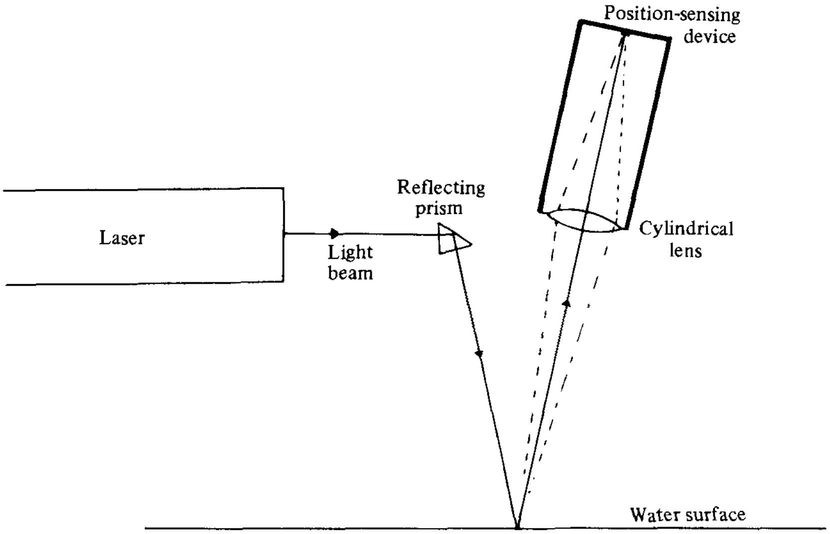


FIGURE 4. The laser-beam apparatus used to measure the variation of surface slope in the Reynolds ridge. The optical system as shown here is arranged to measure beam deflections perpendicular to the plane of the diagram, the cylindrical lens being arranged to bring to a line focus all deflections in the diagram plane. In the arrangement used, the water flow was perpendicular to the plane of the diagram.

5. Results

The results of preliminary direct observations of the Reynolds ridge are shown in figure 1. These pictures were made using a straight-line light source made from a fluorescent light fitting, masked so that only a strip about 1 mm wide was luminescent (Gilbert & Scott 1980). This source was mounted on the flow channel about 60 mm above the water surface, and the two pictures shown in figure 1 were made using a Pentax 35 mm camera (55 mm Takumar lens) pointing alternately downstream and upstream of the ridge. The approximate inversion of the observed image is apparent in these two pictures, and this observation leads to the conclusion that the surface slope variation is of major importance for determining the visual appearance of the ridge (see §2.2).

More-detailed measurements were made by making scans, with the laser beam probe, across the ridges formed at a wide range of stream velocities, as described in §4. Typical scans of surface slope against surface position, for two flow velocities, are shown in figure 5, where they are compared with the predictions of the theory due to Harper & Dixon, using parameter values as follows: $\sigma = 72\ \mathrm{mN\,m^{-1}}$, $g = 9{\cdot}81\ \mathrm{m\,s^{-2}}$, $\rho = 10^6\ \mathrm{g\,mm^{-3}}$ and $\nu = 10^{-6}\ \mathrm{m^2\,s^{-1}}$.

In these figures the signs of the slopes are relative to the flow direction; if the surface rises along this direction then the slope is given as positive. It can be seen that, approaching the ridge from the surface-film-free upstream side the surface first slopes gradually upwards, the slope becoming steeper, before it enters a region of rapidly decreasing slope – a concave-downwards region of high curvature. It is this region of rapidly changing slope which makes the ridge visible through its focusing effect on ambient light.

It is apparent that the agreement between theory and observation is good in the

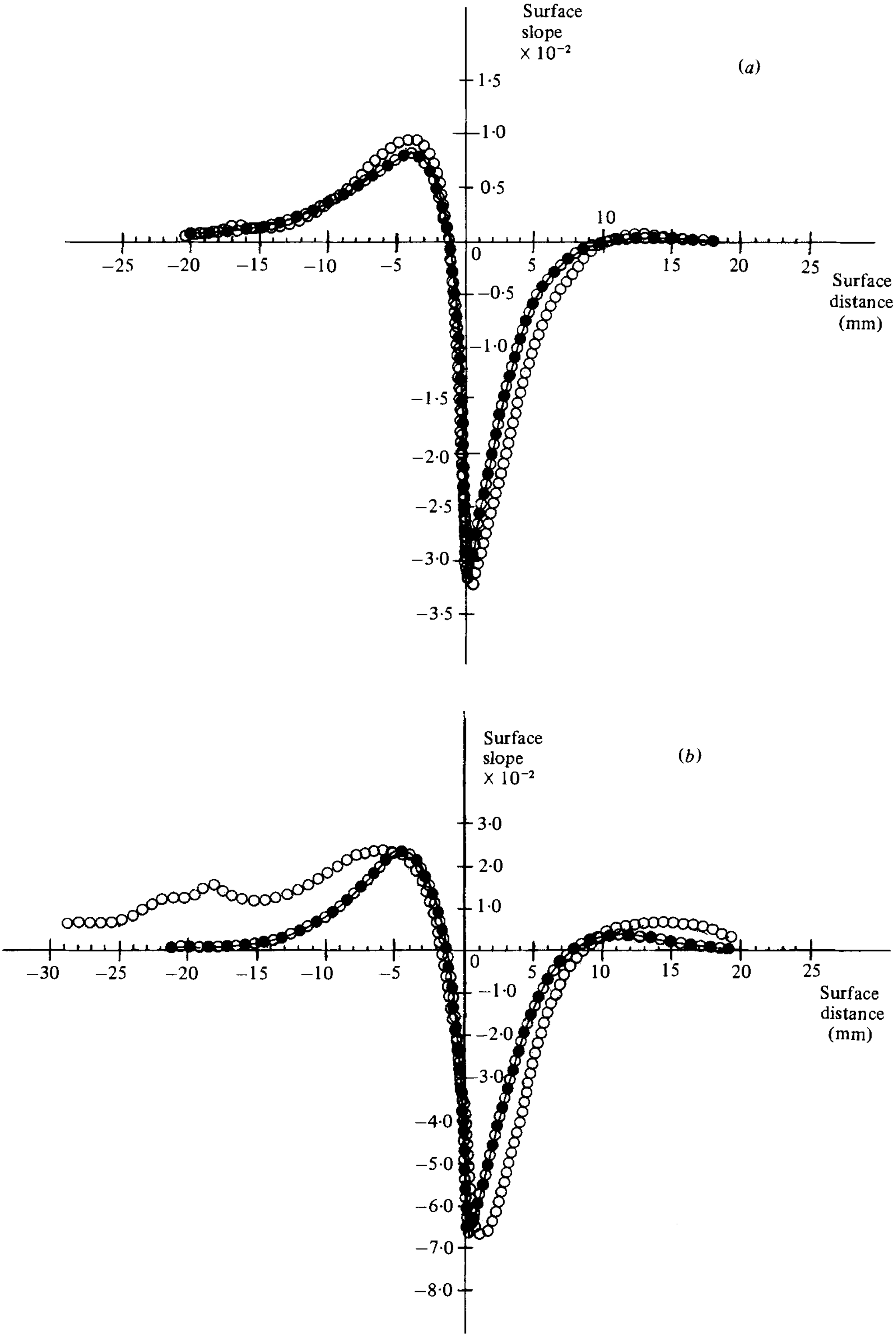


FIGURE 5. The slope variation associated with the Reynolds ridge for two flow velocities: (*a*) 132 mm/s; (*b*) 180 mm/s. The curves drawn with unfilled circles are taken from the oscilloscope traces recorded using the laser-beam scanning apparatus, and the curves with alternate filled and unfilled circles follow the theoretical predictions of Harper & Dixon (1974). Positive slope indicates that the surface rises along the flow direction.

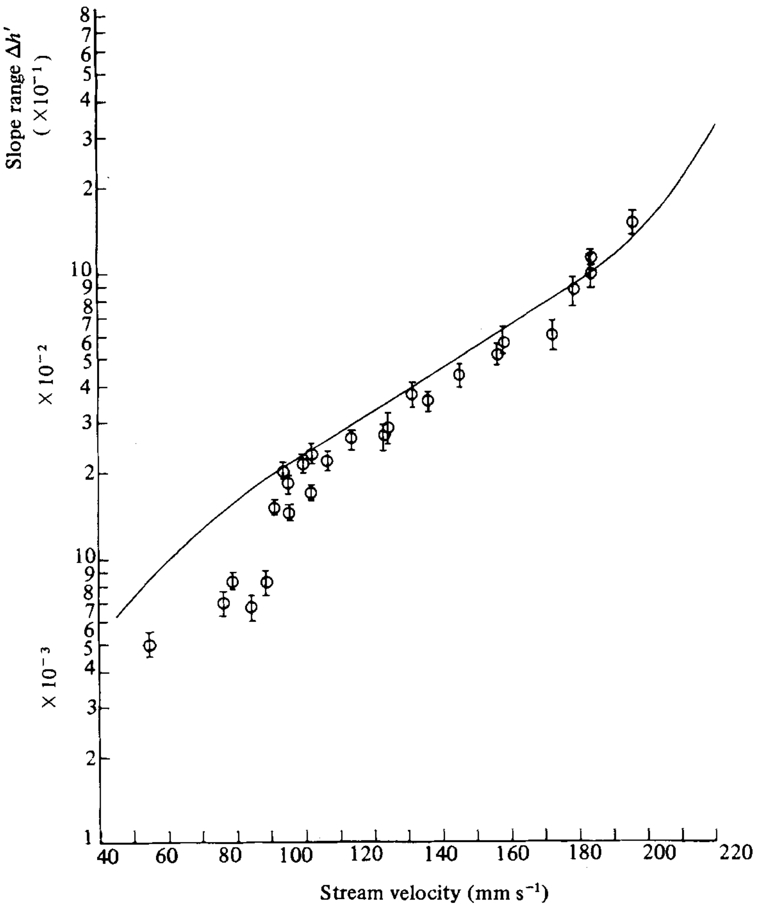


FIGURE 6. Experimental determinations of the slope-range parameter (the difference between the maximum and minimum slopes recorded during a measurement) plotted as a function of surface-flow velocity. The error bars shown represent the mean deviation of the set of measurements made during a particular experimental run at the velocity concerned. The continuous curve shows calculations using the theory of Harper & Dixon (1974).

examples shown, for the two mid-range stream velocities shown. In order to examine the correspondence between theory and experiment over the full range examined experimentally, a suitable representative parameter is needed. The parameter here chosen for this is the range of surface slope involved in the ridge, $\Delta h'$, the difference between maximum and minimum slopes. This parameter has the advantage of being readily measured from the experimental records, and it is also a striking feature of the slope variation, which changes markedly as the flow velocity changes.

The slope-range parameter is plotted in figure 6 as a function of surface-flow velocity, and is compared with the theoretical results. As can be seen, the agreement remains good over a considerable range of velocity. The slope range is plotted on a logarithmic scale, and the ridge is seen to vary enormously in scale over quite a modest range of velocity.

The theoretical curve shown in figure 6 was derived by evaluating the Harper & Dixon theory closely near the peak regions. Agreement with experiment is good, except at very low velocities, where the measured peak-slope difference was less than is predicted. This behaviour would be expected as a result of the residual contamination in the system, the contamination having more time at the lower velocities to diffuse to the surface.

The Harper & Dixon model is seen to be in excellent agreement with observations, and it must therefore be concluded that the shortcomings of the model used, described in §3, are not important. The agreement is best at the highest flow velocities, for which any residual contamination effect would be expected to be least.

6. Conclusions

Measurements were made of the surface-slope variation associated with the Reynolds ridge formed by naturally occurring organic contamination on flowing water, and these results were found to be in good agreement with theoretical results derived by Harper & Dixon (1974). Apparent deficiencies in the theoretical model used by those authors thus appear not to be significant, and it appears that the ridge – one of the most frequently rediscovered phenomena in fluid mechanics – has now been satisfactorily explained.

This work was done while the author was supported by a Fellowship provided by the Natural Environment Research Council. The author currently holds a Fellowship given by the Science Research Council, and he is grateful to both Councils for their support. Thanks are also due to Mr J. K. Bartington, technician of the Fluid Mechanics Research Institute, University of Essex, for assistance with the construction of the apparatus.

REFERENCES

ANON. 1974 *Anal. Chem.* **46**, 799A.

BELL, G. 1954 *Weather*, **9**, 386.

BURDON, R. S. 1926 *Proc. Phys. Soc. Lond.* **38**, 148–160.

BURDON, R. S. 1949 *Surface Tension and the Spreading of Liquids*, p. 68. Cambridge University Press.

EDSER, E. 1926 *General Physics for Students*, 2nd edn, pp. 639–645. Macmillan.

GILBERT, A. D. 1980 *Opt. Acta* **27**, 767–781.

GILBERT, A. D. & SCOTT, J. C. 1980 *Opt. Acta* **27**, 753–766.

HALL, R. O. 1936 *Nature*, **138**, 466.

HARPER, J. F. & DIXON, J. N. 1974 In *Proc. 5th Australasian Conf. on Hydraulics and Fluid Mech., Christchurch, New Zealand*, pp. 499–505.

HOLDER, D. W. & NORTH, R. J. 1963 *Schlieren Methods.* Notes on Appl. Sci. no. 31. London: H.M.S.O.

LANGTON, J. 1872 *Nature* **5**, 241–242.

MCCUTCHEN, C. W. 1970 *Science* **170**, 61–64.

MCDOWELL, R. S. & MCCUTCHEN, C. W. 1971 *Science* **172**, 973.

MERSON, R. L. & QUINN, J. A. 1965 *A.I.Ch.E. J.* **11**, 391–395.

MOCKROS, L. F. & KRONE, R. B. 1968 *Science* **161**, 361–363.

RAYLEIGH, LORD 1890 *Proc. R. Instn* **13**, 85–97.

REYNOLDS, O. 1881 *Brit. Assn Rep.*, pp. 524–525.

SATTERLY, J. 1919 *Trans. R. Soc. Canada* **13**, 109.

SATTERLY, J. 1956 *Am. J. Phys.* **24**, 463.

SATTERLY, J. & TURNBULL, R. 1929 *Trans. R. Soc. Canada* **23**, 95–118.

Schmidt, J. 1936 *Nature* **137**, 777. (See also *Nature* **138**, 20 (1936) for editorial comment.)

Scott, J. C. 1974 *J. Phys.* E*: Sci. Instrum.* **7**, 747–749.

Scott, J. C. 1979 In *Surface Contamination: Its Genesis, Detection and Control* (ed. K. L. Mittal), vol. 1, pp. 477–497. Plenum.

Sellin, R. H. J. 1968 *Nature* **217**, 536–538.

Stansfield, H. 1936 *Nature* **137**, 1073–1074 (See also *Nature* **138**, 20 (1936) for editorial comment).

Thompson, J. C. 1919 *Trans. R. Soc. Canada* **13**, 129–138.

Thoreau, H. D. 1902 (reprinted 1962) *The Journals of Henry D. Thoreau* (ed. B. Torrey & F. H. Allen), vol. 6, pp. 326–327 (1854); vol. 10, p. 256 (1858); vol. 11, pp. 408–409 (1879). Dover.

Woog, P. 1931 *Annales de l'Office Nationale de Combustibles Liquides* **6**, 617–653.